

Algorithms for ocean-bottom albedo determination from in-water natural-light measurements

Robert A. Leathers and Norman J. McCormick

A method for determining the ocean-bottom optical albedo R_b from in-water upward and downward irradiance measurements at a shallow site is presented, tested, and compared with a more familiar approach that requires additional measurements at a nearby deep-water site. Also presented are two new algorithms for estimating R_b from measurements of the downward irradiance and vertically upward radiance. All methods performed well in numerical situations at depths at which the influence of the bottom on the light field was significant. © 1999 Optical Society of America

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1. Introduction

For shallow ocean waters, knowledge of the optical bottom albedo R_b is necessary to model the underwater^{1,2} and above-water³ light field, to enhance underwater object detection or imaging,⁴ and to correct for bottom effects in the optical remote sensing of water depth^{5,6} or inherent optical properties (IOP's).^{7,8} Measurements of R_b can also help one identify the bottom-sediment composition,⁶ determine the distribution of benthic algal or coral communities,⁹ and detect objects embedded in the sea floor. Furthermore, values of R_b , defined as the upward irradiance emerging from the bottom divided by the downward irradiance into the bottom, can be used as an integral test for attempted measurements of the bottom bidirectional reflectance function.

Although the value of the irradiance ratio $R(z)$ equals R_b at the bottom, it is not possible to measure $R(z)$ right at the bottom, and irradiance measurements just above the bottom are difficult to obtain because of instrument self-shadow. An estimate of R_b can be made by extrapolating to the bottom measurements of $R(z)$ at several depths z near the bottom. However, extrapolation is generally unreliable

because profiles of $R(z)$ typically vary sharply with depth close to the bottom.¹⁰ *In vitro* R_b measurements of small bottom samples can be obtained with the method in Ref. 5 or of larger bottom samples with a spectral radiometer. However, these processes are time-consuming. Also the *in vitro* value of R_b is not necessarily equal to the *in situ* value, and it is not clear how representative these samples are of larger spatial regions of interest.

Because *in situ* estimates of R_b from light measurements close to the bottom are subject to small-scale horizontal variability of the bottom, determining R_b from measurements farther from the bottom may be preferable, thereby obtaining horizontally averaged values that are more appropriate for remote-sensing applications and one-dimensional radiative-transfer modeling. R_b can be estimated⁹ from an $R(z)$ measurement just below the sea surface $R(0^+)$ together with simultaneous measurements in nearby deep water of $R(0^+)$ and the downward diffuse attenuation coefficient. Similarly, a qualitative algorithm has been proposed⁶ and tested^{11,12} for bottom characterization from remote radiance measurements at two wavelength bands simultaneously over both shallow and deep water. The disadvantage of both of these methods, however, is that they require a deep-water site nearby that has the same water composition, illumination, and surface conditions as the shallow-water site.

A new method of solving the inverse-radiative-transfer problem for determining R_b is proposed in Section 2. Measurements of the upward and the downward irradiances at one wavelength and at least two mid-water-column depths at only one site are required. Also proposed are two related algorithms

When this research was performed, the authors were with the Department of Mechanical Engineering, University of Washington, Box 352600, Seattle, Washington 98195-2600. R. A. Leathers is now with the NASA Earth System Science Office, Code SA00, Stennis Space Center, Mississippi 39529. His e-mail address is rleather@ssc.nasa.gov.

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for estimating R_b from measurements of the vertically upward radiance and downward irradiance. These algorithms, like the previously developed ones, require knowledge of the measurement distances above the bottom. Results of specific numerical tests for all the R_b estimation algorithms are presented in Section 3, and a discussion is given in Section 4.

2. Theory

A. Preliminaries

We are interested in the azimuthally averaged radiance $L(z, \mu)$ that satisfies the integro-differential radiative-transfer equation:

$$\left(\mu \frac{\partial}{\partial z} + c\right)L(z, \mu) = b \int_{-1}^1 \tilde{\beta}(z, \mu, \mu')L(z, \mu')d\mu', \quad (1)$$

where b and c are the scattering and the beam attenuation coefficients, $\tilde{\beta}$ is the azimuthally integrated scattering phase function, and μ is the direction cosine with respect to the downward depth z . All the quantities in Eq. (1) implicitly depend on wavelength. The downward and upward irradiances are given by

$$\begin{aligned} E_d(z) &= 2\pi \int_0^1 \mu L(z, \mu) d\mu, \\ E_u(z) &= 2\pi \int_{-1}^0 |\mu| L(z, \mu) d\mu. \end{aligned} \quad (2)$$

The irradiance ratio is

$$R(z) = E_u(z)/E_d(z), \quad (3)$$

and $R(z_b) = R_b$ for water depth z_b . The analogous radiance-irradiance ratio is

$$R^L(z) = \pi L_u(z)/E_d(z), \quad (4)$$

where the vertically upward radiance $L_u(z) = L(z, -1)$. The factor π is included in the definition of $R^L(z)$ so that $R^L(z_b) \approx R_b$. For a Lambertian bottom, $R^L(z_b) = R_b$.

With increasing depth in optically deep, spatially uniform, source-free waters, $R(z)$, $R^L(z)$, and the downward diffuse attenuation coefficient,

$$K_d(z) = -\frac{1}{E_d(z)} \frac{dE_d(z)}{dz}, \quad (5)$$

asymptotically approach values R_∞ , R_∞^L , and K_∞ , respectively. These asymptotic values are IOP's of the water that can be uniquely computed from b , c , and $\tilde{\beta}$.¹³

B. Irradiance-Ratio Approach

A well-known model for the irradiance ratio is^{5,6}

$$R(z) = R_{2nd}(z) + [R_b - R_{2nd}(z)]\exp[-2(z_b - z)K_{2nd}], \quad (6)$$

where the water depth at the shallow site of interest is assumed to be known and the subscript $2nd$ denotes measurements taken at a nearby deep-water site characterized by the same illumination, sea-surface conditions, and water IOP's as the site of interest. In the derivation of Eq. (6), K is taken to be $K(z) = [K_d(z) + \kappa(z)]/2$ averaged over depth in a nonspecified manner, where κ is the coefficient of attenuation in the upward direction of upwelling photons^{14,15} (from both in-water scattering and the bottom); a qualitative and quantitative study of κ is given in Refs. 5 and 15. In practice, however, the value of K_{2nd} is typically approximated by the vertically averaged $K_d(z)$ at the deep-water site.⁴ A rearrangement of Eq. (6) gives the algorithm evaluated by Maritorena *et al.*⁵ for determining R_b from irradiance measurements at two sites^{9,16}:

$$R_b = R_{2nd}(z) + [R(z) - R_{2nd}(z)]\exp[2(z_b - z)K_{2nd}]. \quad (7)$$

Equation (7) is often written for $z = 0^+$ in hope of applying it to remote-sensing applications. However, it is valid for any depth z and is more accurate at mid-water depths than at the surface. A difficulty with implementing Eq. (7) is that a deep-water ($2nd$) site may not be available that matches the water, surface, and illumination conditions of the shallow-water site.

An alternative shallow-water model was previously derived¹⁷ from the radiative-transfer equation with the eigenfunction expansion method. In this derivation the light field is approximated by the sum of two eigenmodes that decrease in magnitude with distance away from the surface and bottom. Expressed in the form of Eq. (7), this model is

$$R_b = R_\infty + [R(z) - R_\infty]\exp[2(z_b - z)K_\infty], \quad (8)$$

which can be used to estimate R_b from measurements at a single site provided that z_b is known and R_∞ and K_∞ can be determined. Note that the asymptotic in-water irradiance ratio R_∞ in Eq. (8) does not represent the same quantity denoted by that symbol in Refs. 4 and 9, where R_∞ is our $R_{2nd}(0^+)$. Equations (7) and (8) were derived with entirely independent approaches. However, their final forms are very similar, and the two theoretically converge when R_{2nd} and K_{2nd} are measured at large depths in homogenous, source-free waters, where $R_{2nd}(z) = R_\infty$ and $K_{2nd}(z) = K_\infty$.

Equation (8) provides a new interpretation of the attenuation coefficient in Eq. (7). The derivation of Eq. (7) suggests that $K \approx (K_d + \kappa)/2$, whereas the derivation of Eq. (8) suggests that $K \approx K_\infty$. Because $\kappa > K_d$,^{4,15} and therefore $K > K_d$, there is a question as to the appropriateness of taking K to be the vertically averaged $K_d(z)$. Because typically $\kappa > K_\infty > K_d$, it follows that $K = [K_d(z) + \kappa(z)]_{2nd}/2 \approx K_\infty$. Therefore Eq. (7) may be best implemented by taking K equal to the value of $K_d(z)$ deep in the euphotic zone, where $K_d(z) \approx K_\infty$ rather than as earlier pro-

posed.^{9,16} This hypothesis is further addressed in Section 3.

If R_∞ and K_∞ can be determined at the shallow site of interest, the method of Eq. (8) has the distinct advantage over that of Eq. (7) that measurements are required from only one site. One way to determine R_∞ and K_∞ is to calculate them with the procedure in Ref. 17 from measurements of b and c obtained, for example, from water samples¹⁸ or with a Wetlabs ac-9 instrument.¹⁹ Alternatively, it is possible to estimate R_∞ and K_∞ from the same irradiance-profile measurements (at the shallow-water site) used to form $R(z)$ without any direct measurement of the water properties. The value of R_∞ can be obtained with an equation derived¹⁷ from a shallow-water asymptotic approximation of the light field:

$$\left(\frac{1 - R_\infty}{1 + R_\infty}\right)^2 = \frac{[E_d(z) - E_u(z)]^2|_{z_2}}{[E_d(z) + E_u(z)]^2|_{z_1}}. \quad (9)$$

To employ Eq. (9), one must subtract and add $E_u(z)$ and $E_d(z)$ at two depths, z_1 and z_2 , square the results, and evaluate the differences between the two depths. Because these operations are susceptible to noise, it is important that the irradiance measurements be of high quality and their temporal variations be averaged out. The value of K_∞ in Eq. (8) can be estimated as the maximum value attained by $K_d(z)$. Because the value of $K_d(z)$ is relatively insensitive to R_b ,¹ the value of $\max[K_d(z)]$ is typically approximately equal to K_∞ .

C. Radiance-Irradiance Ratio Approach

If $L_u(z)$ measurements are available rather than $E_u(z)$, R_b can be estimated from $R^L(z)$ of Eq. (4) with a new model (derived in Appendix A) analogous to Eq. (8):

$$R_b = R_\infty^L + [R^L(z) - R_\infty^L] \exp[2(z_b - z)K_\infty]. \quad (10)$$

However, because an equation analogous to Eq. (9) has not been derived for R_∞^L , implementation of Eq. (10) requires that the value of R_∞^L either be calculated¹³ from local measurements of the IOP's or be measured in nearby deep water.

We found with numerical simulations that reasonable estimates of R_b can alternatively be obtained from $R^L(z)$ with

$$R_b = R_{2nd}^L(z) + [R^L(z) - R_{2nd}^L(z)] \exp[2(z_b - z)K_{2nd}], \quad (11)$$

although we have no analytical justification for Eq. (11) other than its analogy to Eq. (7). As with Eq. (7) use of this equation requires measurements at a second (deep-water) site.

3. Numerical Tests

A. Methods

Numerical tests were performed to evaluate the accuracies of Eqs. (7)–(11) for determining R_b . Simulated $E_u(z)$, $E_d(z)$, and $L_u(z)$ values were generated at

0.25 optical depth spacing by using the discrete ordinates radiative-transfer code DISORT.²⁰ The surface illumination was modeled as a combination of direct collimated sunlight and diffuse skylight. The water was defined to have locally homogenous optical properties, a relative index of refraction of 1.34 with respect to air, and scattering determined by the Petzold particle-scattering phase function.²¹ Spatially dependent internal sources, such as from fluorescence, Raman scattering, or bioluminescence, were neglected. A Lambertian bottom was assumed, which provides a good approximation to the more general, but usually poorly known, bidirectional reflectance function.^{1,2} Simulations were performed for various values of the single-scattering albedo ($\omega_0 = b/c$), R_b , percent direct sunlight, and water optical depth ($\tau_b = cz_b$). The values of R_b were taken to be in the range $0 \leq R_b \leq 0.4$, which is consistent with observations in natural waters.¹

Equation (8) was applied to shallow-water simulations of $E_u(z)$ and $E_d(z)$ to determine bottom albedo estimates $\widehat{R}_b(z)$ as a function of the depth of the corresponding irradiance measurements. The values of R_∞ and K_∞ used in Eq. (8) were obtained in two different ways. First, R_∞ was determined with Eq. (9) and K_∞ was estimated from either $K_\infty(z) \approx K_d(z)$ or $K_\infty \approx \max[K_d(z)]$. Second, the IOP's of the water were assumed to be known from *in situ* measurements, and these IOP's were used to calculate R_∞ and K_∞ . For comparison, $\widehat{R}_b(z)$ was also determined from $E_u(z)$ and $E_d(z)$ with Eq. (7) for combinations of shallow and deep-water simulations. Here K was taken to be, alternatively, $K_d(z)$, $\overline{K_d(z)}$ [the vertical average of $K_d(z)$ between the surface and the depth of the shallow-water site], and K_∞ [determined from a deep (asymptotic) value of $K_d(z)$].

The bottom albedo was also determined from simulations of $L_u(z)$ and $E_d(z)$. Equation (10) was used to calculate $\widehat{R}_b(z)$ from data at only the shallow-water site. The values of R_∞^L and K_∞ were calculated from the known water optical properties. In addition $\widehat{R}_b(z)$ was determined from $L_u(z)$ and $E_d(z)$ with Eq. (11) for combinations of shallow- and deep-water sites. The value of K_{2nd} was determined in the same manner as for the $E_u - E_d$ approach.

B. Results

For all the shallow-water simulations performed, estimates of R_b obtained with Eqs. (8) and (9) approached the correct value in a nearly linear fashion within the bottom few optical depths of the water column. Extrapolation of $\widehat{R}_b(z)$ at two and one optical depths above the bottom consistently produced estimates of R_b that were accurate to within $\sim 1\%$. In Table 1 example $\widehat{R}_b(z)$ is given at three, two, and one optical depths above the bottom as well as the linear extrapolation of the latter two to the bottom. For Table 1 the illumination conditions of the simulations were taken to be either overcast (100% diffuse) or sunny (75% direct sunlight from a zenith

Table 1. Calculations of the Bottom Albedo^a

Simulation				$\widehat{R}_b(\tau)$			
ω_0	τ_b	R_b	Illumination	$\tau_b - 3$	$\tau_b - 2$	$\tau_b - 1$	Extrapolation
0.7	3	0.2	Sunny	N/A	0.177	0.189	0.200
0.7	5	0.1	Sunny	0.091	0.091	0.096	0.101
0.7	5	0.2	Sunny	0.170	0.176	0.189	0.202
0.7	5	0.2	Overcast	0.172	0.177	0.190	0.202
0.7	7	0.2	Sunny	0.167	0.176	0.189	0.203
0.8	5	0.2	Sunny	0.186	0.188	0.194	0.201
0.9	5	0.1	Sunny	0.114	0.106	0.102	0.098
0.9	5	0.2	Sunny	0.207	0.201	0.200	0.199
0.9	5	0.2	Overcast	0.207	0.201	0.200	0.199
0.9	5	0.3	Sunny	0.296	0.294	0.297	0.300
0.9	5	0.4	Sunny	0.385	0.387	0.395	0.403

^aCalculations are with Eqs. (8) and (9) from simulated irradiance measurements at three, two, and one optical depths above the bottom and extrapolations to the bottom of the latter two.

angle of 30°), and in the solutions for R_b the values of K_∞ in Eq. (8) were approximated by $\max[K_d(z)]$. At a given measurement depth estimates generally improved with the increasing value of ω_0 . For example, for $R_b = 0.2$, $\tau_b = 5$, and sunny skies, the error at three optical depths above the bottom was 15% for $\omega_0 = 0.7$ but only 3.5% for $\omega_0 = 0.9$. The value of R_b had a small effect on the accuracy of its estimate. For large ω_0 and small R_b , $\widehat{R}_b(z)$ was greater than R_b , whereas for small values of ω_0 or for a combination of large ω_0 and large R_b , $\widehat{R}_b(z)$ was less than R_b . Estimates were more accurate, but insignificantly so, with overcast conditions. Also, the depth of the water had little effect on the accuracy of $\widehat{R}_b(z)$ at a given depth above the bottom. However, in practice, instrument noise will be more significant in relatively deep water than in shallow water.

In most cases, estimates of R_b with Eqs. (8) and (9) were more accurate if K_∞ in Eq. (8) was approximated by $\max[K_d(z)]$ than if it was replaced by $K_d(z)$, because large values of K_∞ in Eq. (8) lead to large values of \widehat{R}_b , and the estimates of R_b were typically less than the correct value. For cases in which the value of ω_0 was high and the value of R_b small, the use of $K_d(z)$ gave slightly, but insignificantly, better results than the use of $\max[K_d(z)]$.

Because Eq. (9) provides only an asymptotic approximation to R_∞ , it was expected that estimates of R_b would improve if more accurate values of R_∞ , calculated from the assumed known water IOP's, were used in Eq. (8). However, the numerical tests showed the reverse to be true; errors in $\widehat{R}_b(z)$ introduced by the approximation of Eq. (9) helped counteract errors in $\widehat{R}_b(z)$ due to the assumptions inherent in Eq. (8). Although Eq. (8) with a calculated R_∞ performed similarly in the bottom half of the water column to Eq. (8) with R_∞ from Eq. (9), Eq. (8) with a calculated R_∞ performed poorly in the top half of the water column and even near the bottom slightly underperformed Eqs. (8) and (9).

For example, shown in Fig. 1 are the estimates of

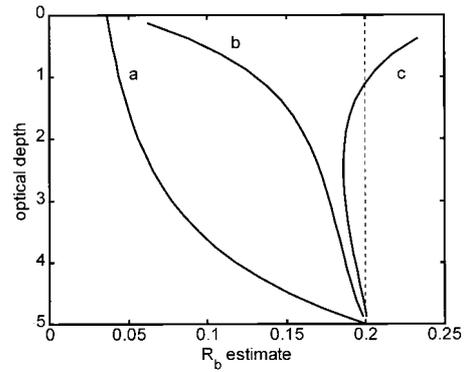


Fig. 1. Profiles of estimates of the bottom albedo $\widehat{R}_b(\tau)$ with, a, the estimate equal to the irradiance reflectance $R(\tau)$; b, Eq. (8) with known R_∞ and K_∞ ; and c, Eqs. (8) and (9). The simulation was generated for $R_b = 0.2$, five optical depth water, $\omega_0 = 0.8$, and 75% collimated light at 30° from the zenith.

R_b as a function of the measurement optical depth obtained from simulated $E_u(z)$ and $E_d(z)$ with three different methods: from $\widehat{R}_b(z) = R(z)$, from Eq. (8) with calculated R_∞ and K_∞ , and from Eqs. (8) and (9) with K_∞ replaced by $\max[K_d(z)]$. In this simulation $R_b = 0.2$, $\tau_b = 5$, and $\omega_0 = 0.8$, and the sea-surface illumination was taken to be sunny (as defined above). Since $R_b = R(z \rightarrow z_b)$, the first approach for determining R_b is the most straightforward. This approach gave a smooth profile of $\widehat{R}_b(\tau)$ that monotonically approached R_b with increasing depth. However, this estimate is extremely inaccurate except very close to the bottom, with 41% error at only one optical depth above the bottom and 61% error at two optical depths above the bottom. This sharp increase in $R(z)$ near the bottom is typical¹⁰ and makes extrapolation of $R(z)$ from mid-water depths to the bottom impractical. At all depths off the bottom, far better estimates of R_b were obtained with Eq. (8) with R_∞ and K_∞ calculated from the known water IOP's. The errors at two and one optical depths off the bottom were 11% and 5.8%, respectively. Even better estimates of R_b at all depths off the bottom, however, were obtained from Eq. (8) with R_∞ determined with Eq. (9) and K_∞ estimated by $\max[K_d(z)]$. This method gave an error at two and one optical depths off the bottom of 6.2% and 2.8%, respectively. Again, extrapolation from mid-depths gave an excellent estimate of R_b .

Values of the bottom albedo from irradiance measurements at a combination of shallow- and deep-water sites with Eq. (7) were consistently underestimated. Therefore, since K_∞ was generally larger than $K_d(z)$, results were more accurate when the value of K in Eq. (7) was determined from a deep value of $K_d(z)$ in the deep-water site, where $K_d(z) \approx K_\infty$, than when it was determined from either the deep-water $K_d(z)$ or $\overline{K_d(z)}$. For example, shown in Fig. 2 are $\widehat{R}_b(\tau)$ calculated from Eq. (7) with K replaced by $K_d(z)$ and by K_∞ [determined from $K_d(\tau = 15)$] for two sunny-sky simulations: (1) $\omega_0 = 0.7$, τ_b

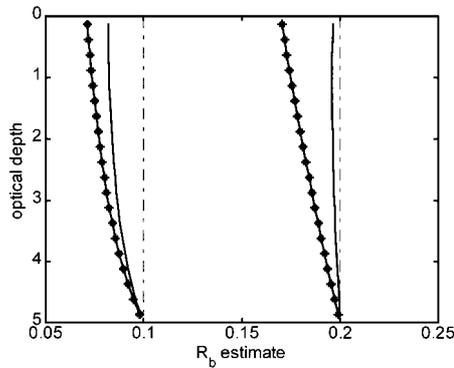


Fig. 2. Profiles of estimates of the bottom albedo $\widehat{R}_b(\tau)$ with the two-site $E_u - E_d$ method of Eq. (7) (left) for $\omega_0 = 0.7$ and $R_b = 0.2$ and (right) for $\omega_0 = 0.9$ and $R_b = 0.1$. The simulations were for sunny conditions, and the R_b determination was done with K in Eq. (7) replaced ($- * -$) by $\widehat{K}_d(z)$ and ($-$) by the deep (asymptotic) value of $K_d(z)$.

= 5, and $R_b = 0.1$ and (2) $\omega_0 = 0.9$, $\tau_b = 5$, and $R_b = 0.2$.

Shown in Fig. 3 are comparisons of $\widehat{R}_b(\tau)$ obtained from $E_u(z)$ and $E_d(z)$ measurements at only the shallow-water site [Eqs. (8) and (9)] and $\widehat{R}_b(\tau)$ obtained from measurements at both shallow- and deep-water sites [Eq. (7)]. The two cases shown are sunny-sky simulations with $\tau_b = 5$ and (1) $\omega_0 = 0.9$ and $R_b = 0.1$ and (2) $\omega_0 = 0.8$ and $R_b = 0.2$. As these examples demonstrate, the two-site method was typically far more accurate than the one-site method near the surface, whereas the one-site algorithm usually outperformed the two-site method near the bottom.

Example $\widehat{R}_b(\tau)$ obtained from $L_u(z)$ and $E_d(z)$ are shown in Fig. 4 for (1) $\omega_0 = 0.9$, $R_b = 0.1$, and sunny conditions and (2) $\omega_0 = 0.9$, $R_b = 0.2$, and overcast conditions. Because estimates of R_b with Eq. (11) were typically larger than the actual value, $\widehat{R}_b(\tau)$ in Fig. 4 obtained with Eq. (11) was calculated with $\widehat{K}_d(z)$ rather than the asymptotic K_d to make the

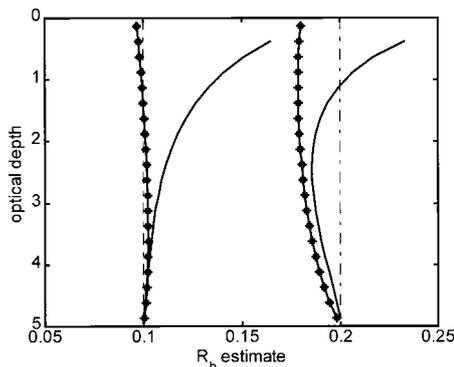


Fig. 3. Profiles of estimates of the bottom albedo $\widehat{R}_b(\tau)$ from $E_u(z)$ and $E_d(z)$ with ($- * -$) the two-site method of Eq. (7) and ($-$) the one-site method of Eq. (8). These sunny-sky simulations were (left) for $\omega_0 = 0.9$ and $R_b = 0.1$ and (right) for $\omega_0 = 0.8$ and $R_b = 0.2$.

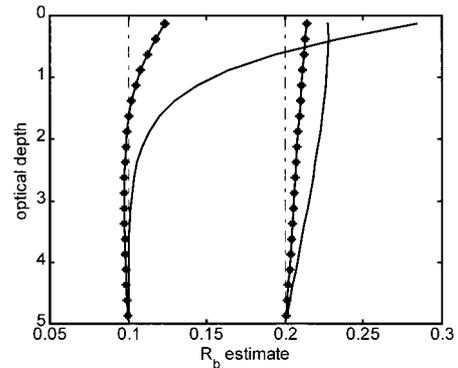


Fig. 4. Profiles of estimates of the bottom albedo $\widehat{R}_b(\tau)$ from $L_u(z)$ and $E_d(z)$ with ($- * -$) the two-site method of Eq. (11) and ($-$) the one-site method of Eq. (10). The simulations were (left) for sunny conditions, $\omega_0 = 0.9$ and $R_b = 0.1$, and (right) for overcast conditions, $\omega_0 = 0.9$ and $R_b = 0.2$.

estimated values as good as possible. In general the value of ω_0 had a significant effect on the accuracy of both $L_u - E_d$ methods, with the best estimates obtained when ω_0 was large. The illumination conditions were also very important for the $L_u - E_d$ methods, with the best results obtained for overcast conditions. The value of R_b , on the other hand, had no significant effect on the accuracy of its estimate. In sunny conditions the one-site method of Eq. (10) and the two-site method of Eq. (11) performed similarly in the bottom one optical depth when ω_0 was small and in the bottom three optical depths when ω_0 was large, but Eq. (11) was considerably more accurate than Eq. (10) near the surface. For overcast conditions Eq. (10) performed well at all depths but still underperformed Eq. (11).

4. Discussion

Several methods were evaluated here for determining R_b from common natural-light measurements. Each method returns accurate values of R_b if implemented close to the bottom. However, because it is difficult in practice to obtain light-field measurements close to the bottom, it is necessary to apply these algorithms at one or more optical depths off the bottom and, when possible, extrapolate the depth-dependent estimates to the bottom. Therefore it is desirable that the error of the method used be both small and linearly decreasing with depth. The methods were all found to be preferable to a straightforward extrapolation of $R(z)$ to the bottom, where $R(z) = R_b$, but differed in their accuracies when applied more than one or two optical depths away from the bottom.

Estimates of R_b can be obtained with Eq. (8) first by estimating R_∞ either with Eq. (9) or by calculating it from known water IOP's. This method does not require measurements at other wavelengths or at another site, and estimates of R_b from this method at one or two optical depths off the bottom were generally found to be more accurate than those obtained with Eq. (7). Numerical simulations indicated that

the use of R_∞ from Eq. (9) produces better estimates of R_b than the use of the value of R_∞ computed from the IOP's, because fortuitously the error introduced by applying Eq. (8) where the light field is not well described by the assumed two-mode asymptotic model (see Appendix A) is mitigated by the deviation in the value of R_∞ predicted by Eq. (9) from its true value. Therefore, even if the water IOP's are known, employing Eq. (9) is preferable, provided the processed data have a relatively smooth $\widehat{R}_b(\tau)$ profile. Unfortunately this method is often inaccurate near the sea surface when the bottom signal is not strong.

If one wishes to estimate R_b from measurements close to the surface and a suitable deep-water site is available, Eq. (7) provides the most reliable method. However, it was found that if the deep-water site is vertically well mixed, Eq. (7) should be implemented by replacing K_{2nd} with K_∞ , which can be directly measured deep in the euphotic zone of the deep-water site.

Estimates of R_b alternatively can be made from measurements of $L_u(z)$ and $E_d(z)$ with Eq. (10), provided that the bottom is approximately Lambertian. Equation (10) requires that R_∞^L either be analytically computed from local measurements of the water IOP's or be measured at a nearby deep-water site. If a suitable second site is readily available, Eq. (11) should be used instead since it was found to be generally more accurate and reliable than Eq. (10). However, both $L_u - E_d$ methods performed well when applied in the bottom half of the water column. The inaccuracy of Eq. (10) in sunny conditions makes it unsuitable for remote-sensing applications, and therefore Eq. (10) offers no advantage over the $E_u - E_d$ method (which is the more natural approach to an *in situ* R_b estimation). Equation (11), on the other hand, shows some promise for remote-sensing applications for large ω_0 (or for any ω_0 if ω_0 is known).

Regardless of the method used, the determination of R_b is easiest when the bottom signal is strong. Thus R_b can be obtained most accurately when the water is shallow, the value of R_b is large, and the attenuation of the water is low (e.g., over tropical coral reefs or white sandy beaches). If the bottom composition is believed to be uniform over a large horizontal region, the determination of R_b should be made at the shallowest depth.

In practice the method to use for estimating R_b will be dictated by the instrumentation available and whether an appropriate deep-water site exists. Given the choice, however, the estimation of R_b should be made with measurements of $E_u(z)$ rather than with $L_u(z)$. The most informative approach would be to use all the methods discussed here and intercompare the results. If the estimates agree, they can be recorded with great confidence. On the other hand, the difference between estimated R_b values from $R(z)$ and $R^L(z)$ might serve as a crude measure of the degree to which the bottom behaves as a Lambertian surface. However, for purposes of modeling the light field, Mobley²² has demonstrated that

measuring the magnitude of the effective bottom albedo is far more important than obtaining the detailed angular pattern of the bottom bidirectional reflectance function.

Appendix A: Derivation of Eq. (10)

If the optical properties of water are spatially uniform and there are no internal sources, when z is at least one optical depth away from a boundary the upward radiance and downward irradiance can be expressed as summations of eigenmodes¹³:

$$\begin{aligned} L_u(z) &= L(z, -1) \\ &= \sum_{j=1}^J [C(v_j)\phi(v_j, -1)\exp(-cz/v_j) \\ &\quad + C(-v_j)\phi(-v_j, -1)\exp(cz/v_j)], \end{aligned} \quad (A1)$$

$$\begin{aligned} E_d(z) &= \sum_{j=1}^J [C(v_j)\tilde{g}_1(v_j)\exp(-cz/v_j) \\ &\quad + C(-v_j)\tilde{g}_1(-v_j)\exp(cz/v_j)], \end{aligned} \quad (A2)$$

where $C(\pm v_j)$ are expansion coefficients, v_j are the J eigenvalues of Eq. (1) corresponding to the eigenfunctions²³ $\phi(\pm v_j, \mu)$, and

$$\tilde{g}_j(\pm v_1) = \int_0^1 \phi(\pm v_j, \mu) P_j(\mu) d\mu \quad (A3)$$

for Legendre polynomial $P_j(\mu)$. Far from the boundaries, $E_d(z)$ and $L_u(z)$ can be approximated by retaining only the asymptotic decreasing eigenmode corresponding to the largest eigenvalue v_1 [i.e., $C(v_j) = 0$ for $j > 1$ and $C(-v_j) = 0$ for all j], and since $R^L(z) = \pi L_u(z)/E_d(z)$ the asymptotic value of $R^L(z)$ is $R_\infty^L = \pi\phi(v_1, -1)/\tilde{g}_1(v_1)$. At depths far from the surface but where some influence of the bottom is present, $E_d(z)$ and $L_u(z)$ are better approximated by also including the eigenmode corresponding to the largest negative eigenvalue, $-v_1$, so that

$$\begin{aligned} L_u(z) &\approx C(v_1)\phi(v_1, -1)\exp(-cz/v_1) \\ &\quad + C(-v_1)\phi(-v_1, -1)\exp(cz/v_1), \end{aligned} \quad (A4)$$

$$\begin{aligned} E_d(z) &\approx C(v_1)\tilde{g}_1(v_1)\exp(-cz/v_1) \\ &\quad + C(-v_1)\tilde{g}_1(-v_1)\exp(cz/v_1). \end{aligned} \quad (A5)$$

After forming the ratio $R^L(z) = \pi L_u(z)/E_d(z)$, dividing through by $C(v_1)\tilde{g}_1(v_1)\exp(-cz/v_1)$, letting $r = C(-v_1)/C(v_1)$, and recognizing that¹³ $R_\infty^L = \phi(-v_1, 1)/\tilde{g}_1(v_1)$, we find that

$$R^L(z) = \frac{R_\infty^L + \pi r \phi(-v_1, -1)\exp(2cz/v_1)/\tilde{g}_1(v_1)}{1 + r R_\infty \exp(2cz/v_1)}. \quad (A6)$$

Subtraction of R_∞^L from Eq. (A6) gives

$$\begin{aligned} R^L(z) - R_\infty^L &= \\ &= \frac{r \exp(2cz/v_1)[\pi\phi(-v_1, -1)/\tilde{g}_1(v_1) - R_\infty^L R_\infty]}{1 + r R_\infty \exp(2cz/v_1)}. \end{aligned} \quad (A7)$$

A rearrangement of Eq. (A7) is

$$[R^L(z) - R_\infty^L] \exp(-2cz/v_1) [1 + rR_\infty \exp(2cz/v_1)] \\ = r[\pi\phi(-v_1, -1)/\bar{g}_1(v_1) - R_\infty^L R_\infty]. \quad (\text{A8})$$

Since the right-hand side of Eq. (A8) is independent of z , the left-hand side at arbitrary depth z equals that at the bottom. Therefore

$$R^L(z) = R_\infty^L + [R^L(z_b) - R_\infty^L] \exp[-2c(z_b - z)/v_1] \\ \times \left[\frac{1 + rR_\infty \exp(2cz_b/v_1)}{1 + rR_\infty \exp(2cz/v_1)} \right]. \quad (\text{A9})$$

If $[rR_\infty \exp(2cz_b/v_1)] \ll 1$ or if $(z_b - z)$ is small, Eq. (A9) reduces to

$$R^L(z) = R_\infty^L + [R^L(z_b) - R_\infty^L] \exp[-2c(z_b - z)/v_1], \quad (\text{A10})$$

which is analogous to our equation for $R(z)$ derived in a similar manner.¹⁷ Rearrangement of Eq. (A10) gives Eq. (10).

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